The Logical Structure of Asymmetric Marriage Systems

Per Hage & Frank Harary, *The Logical Structure of Asymmetric Marriage Systems.*—The logical structure of asymmetric marriage systems is examined with respect to four questions. (1) To what extent can transitivity (hierarchy) be neutralized through the formation of cycles? (2) How are transitive structures generated in cyclical systems? (3) How can systems be compared with respect to their degree of transitivity? (4) Are hypergamous systems necessarily non-prescriptive?

Parkin (1990) has proposed that the forms of hierarchy associated with prescriptive and non-prescriptive asymmetric marriage systems can be distinguished on the basis of their logical structure. In non-prescriptive systems, which by definition do not specify or institutionalize the choice of spouse, hierarchy has the form of a “ladder” and is transitive. In prescriptive systems—in Lévi-Strauss’s (1969) terminology “generalized exchange” or in Needham’s (1987: 134) characterization, systems “constructed on the module of marriage with the mother’s brother’s daughter”—hierarchy has the form of a “circle” and is said to be intransitive. The implication is that generalized exchange is not inherently unstable as Lévi-Strauss maintains. Although few if any asymmetric prescriptive systems or systems of generalized exchange are actually intransitive, Parkin’s analysis raises four important questions concerning their structure:

(1) To what extent can transitivity be neutralized through the formation of cycles?

(2) How are transitive structures generated in cyclic systems?

(3) How can different systems be compared with respect to their “degree of transitivity”?

(4) Are hypergamous systems necessarily non-prescriptive?

Parkin’s Models

In systems of asymmetric marriage alliance there are two logically possible forms of status difference between alliance groups: either wife-takers are superior to wife-givers (a relation of hypergamy) or wife-givers are superior to wife-takers (a relation of hypogamy). From a purely formal point of view these are alternative forms of anisogamy and it is not necessary to distinguish between them (Lévi-Strauss 1969, 1987). For Parkin however, these two forms are not simply “mirror images” of each other since they are associated with important ethnographic differences. The situation in which wife-takers are superior to wife-givers is exemplified by the non-prescriptive affinal alliance systems of North India while that in which wife-givers are superior to wife-takers is exemplified by the asymmetric prescriptive systems of Southeast Asia and Eastern Indonesia. Systems of the latter type are found in such well-known societies as the Kachin of North-East Burma (Leach 1954, 1961), the Purum (Needham 1958, 1962) on the Indo-Burma border, and the Mamboru (Needham 1987) located in northwestern Sumba in the Sunda islands of Eastern Indonesia.

In the North Indian (hypergamous) situation status is, ideologically, determined by a relation of caste, sub-caste or sub-sub-caste purity. One way of improving status is by giving unsolicited gifts to ritual superiors. The preeminent gift is that of a virgin accompanied by a dowry. In this case marriage “follows hierarchy.” The hierarchy has the form of a “ladder” and is transitive: “if group A is superior to group B, and group B superior to group C, then group A is necessarily superior to group C” (Parkin 1990: 475). Bottlenecks may occur with a surplus of women at the top and men at the bottom. The solutions include symmetric alliances among low status groups and female infanticide and polygyny among high status groups. Marriages in this system are not repeated in successive generations.

In the Southeast Asian and Eastern Indonesian (hypogamous) situation marriages are repeated in successive generations. With no other basis for status distinctions marriage alliance “creates” the hierarchy. According to Parkin this hierarchy is “intransitive since status differences do not accumulate towards an apex”. The formal model of the system is three or more groups marrying in a circle. Potential transitivity is neutralized through the formation of circles (cycles): “if there are only three groups A, B, and C then A may be wife-givers and therefore superior to B, and B wife-givers and therefore superior to C; but this does not make A superior to C—quite the reverse, because C are A’s own wife-givers...” (Parkin, ibid.: 476). The multiplicity of cycles in an alliance system “mitigates or suppresses entirely the tendency for status differences to accumulate into the open-ended and transitive ladder typified by North India” (ibid.). Parkin claims that status differences can be ren-
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...dered “completely intransitive ultimately”. This is a hierarchy with “no identifiable top or bottom”. It has no bottlenecks since everyone can find a spouse.

In some societies, according to Parkin, transitive and intransitive hierarchies may co-exist. In Mamboru, for example, the alliance system is said, by Needham, to be intransitive but the class system (consisting of chiefs, commoners and serfs/slaves) is transitive. Conflict may arise as when a chief marries a commoner. In other societies the two forms may co-exist leading to conflict in the mind of the ethnographer. Thus in Kachin (gumsa) society according to Leach, the lineage system is internally stratified (transitive) and so is the alliance system since wife-givers (mayu) rank higher than wife-takers (dama) in the lineage hierarchy. Although the Kachin origin myth describes five clans marrying in a circle (as depicted in Fig. 46 in Lévi-Strauss 1969), Leach insisted that in reality marriage is, with one notable exception, noncyclical. The exception is the “cousin-circle path” at the top of the hierarchy: “A chief in order to maintain status, must marry—as his first wife—a woman from another chiefly lineage, that is to say a woman from some other chiefly domain. Such marriages can form the basis of large-scale political alliances. It is not uncommon to find three neighboring chiefly groups A, B, and C, linked by the rule that chief A marries princess B, chief B marries princess C, chief C marries princess A. Such a system is called by the Kachins ‘cousin circle path’ (hkau wang hku). The three chiefs are all of equal status” (Leach 1961: 84-85). Parkin, however, argues on logical and empirical grounds that Kachin marriage is more cyclical than Leach recognized: he adduces two marriage cycles in Leach's own data and he points out that bottlenecks would necessarily occur if such cycles were lacking.

The North Indian situation is clear enough but the Southeast Asian-Eastern Indonesian situation is not. The Purum and Mamboru alliance systems, contrary to Parkin's, and in the case of Mamboru, Needham's assertions are not intransitive or perfectly cyclical and neither is the Kachin five clan system. Neither are most asymmetric prescriptive systems. This fact has interesting cognitive and evolutionary implications. Before proceeding we require a few essential definitions (from Hage & Harary 1991).

Definitions

A relation $R$ is a set of ordered pairs $(u, v)$ of elements from some set $V$, which we specify as finite. When $(u, v)$ is in $R$ we write $uRv$. A relation $R$ is represented graphically by taking the elements of $V$ as nodes and drawing an arc (directed edge) from $u$ to $v$ whenever the ordered pair $(u, v)$ is in $R$ and also an arc from $u$ to itself, called a loop if $(u, u)$ is in $R$.

1. Fig. 13 in Leach (1961) actually shows cousin circle paths at all levels of (gumsa) Kachin society—chiefly, aristocratic and commoner.
(1) Relation \( R \) is reflexive denoted by code letter \((r)\) if there is a loop at every node; it is irreflexive \((\bar{r})\) if no node has a loop. A digraph \( D \) (directed graph) is defined as an \( r \) relation, as illustrated in Fig. 1. We say that a digraph has \( n \) nodes and \( m \) arcs. All three digraphs in Fig. 1 have \( n = 3 \) nodes. The first digraph has \( m = 2 \) arcs while the other two digraphs have \( m = 3 \) arcs each.

(2) Whenever there are two arcs of the form \((u, v), (v, u)\) they form a symmetric pair of arcs. Then \( R \) is symmetric \((s)\) if every arc (other than a loop) is part of a symmetric pair; \( R \) is asymmetric \((\bar{s})\) if no arc is part of a symmetric pair. All three digraphs in Fig. 1 are \( \bar{s} \).

(3) A 2-path from \( u \) to \( v \) consists of two arcs of the form \((u, w)\) and \((w, v)\) with the three nodes \( u, v, w \) distinct. Relation \( R \) is transitive \((t)\) if whenever there is a 2-path from \( u \) to \( v \), the arc \((u, v)\) is also in \( R \). An intransitive \((\bar{t})\) relation is one that is never transitive, that is, whenever there is a 2-path from \( u \) to \( v \) the arc \( uv \) cannot be present. The first two digraphs in Fig. 1 are \( \bar{t} \) while the third is \( t \). A (directed) cycle consists of a nontrivial path together with an arc from its terminal to its initial node. The second digraph, a 3-cycle, is called a cyclic triple while the third digraph is a transitive triple.

(4) Relation \( R \) is complete \((c)\) if for each pair of nodes \( u, v \) of \( R \) at least one of the two arcs \((u, v)\) and \((v, u)\) must occur. A digraph which is \( \bar{r}, \bar{s}, c \) is called a tournament. Figures 1b and 1c display the two tournaments on three nodes.

\[ \begin{array}{c}
\text{(a)} & \text{(b)} & \text{(c)} \\
\end{array} \]

*Fig. 1. Intransitive and transitive digraphs.*

**Analysis**

Notice with respect to the definitions just given, that intransitivity cannot be the uniquely defining characteristic of an asymmetric marriage system: a cycle is intransitive, but so is a digraph having no cycles at all, i.e., a path or more generally a tree. The significant distinction in the analysis of asymmetric prescriptive marriage systems or systems of generalized exchange is between transitive and cyclic structures\(^2\).

\(^2\) The pecking order is sometimes, inaccurately, characterized as intransitive. According to Landau (1951) the pecking order is usually transitive but not always. Occasionally a flock may contain a cyclic triple but this is a rare phenomenon in pecking orders of hens; most pecking orders are transitive and all are complete and asymmetric.
1. Neutralization of transitivity. According to Parkin potentially transitive relations are neutralized through the creation of 3-cycles: in an asymmetric prescriptive system, if A is a wife-giver to B, and B is a wife-giver to C, then the relation of C as a wife-giver to A precludes the relation of A as a wife-giver to C. It must be pointed out, however, that it will often be impossible to eliminate transitivity completely. Let us take the Kachin-type case first. Fig. 2 shows tournaments with three, four and five nodes. The first tournament, denoted $T_3$, consists of a cyclic triple while the second tournament, $T_4$, contains two cyclic and two transitive triples. The third tournament, $T_5$, which is isomorphic to the “feudal cycle of Kachin marriage” (as depicted in Fig. 46 in Lévi-Strauss 1969), contains just five cyclic triples and exactly the same number of transitive triples. Five is in fact the maximum number of cyclic triples in any tournament with five nodes. There is no need to verify this by experimentation since the well known Theorem 1 below, due to Kendall & Smith (1940), gives the maximum number for all tournaments with a given number $n$ of nodes.

The number of arcs in a tournament with $n$ nodes is the binomial coefficient $\binom{n}{2}$ called “$n$ choose 2” given by

$$\binom{n}{2} = \frac{n(n - 1)}{2}.$$ 

The total number of triples, including both transitive triples and cyclic triples, in a tournament with $n$ nodes is

$$\binom{n}{3} = \frac{n(n - 1)(n - 2)}{6}.$$ 

Thus tournaments with three, four and five nodes have one, four and ten triples respectively.

**Theorem 1.** Among all tournaments with $n$ nodes the maximum number of cyclic triples, denoted $c_{\text{max}}(n)$, is

$$c_{\text{max}}(n) = \begin{cases} 
\frac{n^3 - n}{24} & \text{if } n \text{ is odd} \\
\frac{n^3 - 4n}{24} & \text{if } n \text{ is even}
\end{cases}.$$ 

Thus tournaments with three, four and five nodes contain a maximum of one, two and five cyclic triples respectively.

3. There are in fact interesting ethnographic cases in which status differences or relations of indebtedness are deliberately neutralized through the formation of 3-cycles. See for example Whitten (1976: 114) as described in Hage & Harary (1983: 73-74) and Godelier (1982) as discussed in Hage & Harary (1991: 7-9).

4. Technically, these are all “strong” tournaments as they have a spanning cycle which passes through every node. See Harary, Norman & Cartwright (1965) and Hage & Harary (1991).
It follows from Theorem 1 that for large tournaments, the maximum possible proportion of cyclic triples approaches the limit of $1/4$ as the number $n$ of nodes approaches infinity. While most marriage alliance digraphs are not tournaments, any such digraph that contains a tournament with four nodes ($T_4$) cannot be perfectly cyclic but must contain at least two transitive triples. Both the Purum and Mamboru digraphs, implicit in Tables 6 and 15 of Needham (1962) and (1987), have $T_4$ as subgraphs.

Since most asymmetric prescriptive systems are not tournaments it is natural to ask the following question: what is the maximum number of cyclic triples in the digraph of any asymmetric prescriptive system? The answer is given by generalizing the classical result of Kendall and Smith. The result of Theorem 1 can be worded to say that it gives the exact maximum possible number of cyclic triples ($c_{\text{max}} (n)$) in an asymmetric digraph $D$ (also called an oriented graph) with $n$ nodes and $m = \binom{n}{2} = n(n-1)/2$. This number of arcs tells us that $D$ is a tournament. Now we investigate the determination of $c_{\text{max}} (n)$ when an oriented graph of $n$ nodes has an arbitrary number of arcs which may be fewer than $\binom{n}{2}$. We illustrate for $n = 4$ and 5, i.e. oriented graphs with four and five nodes.

First, when $n = m = 3$ (three nodes and three arcs) we have the oriented graph (a tournament, in fact a cyclic triple) in Fig. 3.
When \( n = 4 \) nodes we have result shown in Table 1. (In Table 1 \( m \) starts with 3 as a cyclic triple has three arcs.) This is illustrated in Fig. 4 for a digraph with four nodes and five arcs.

<table>
<thead>
<tr>
<th>( m )</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
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<tbody>
<tr>
<td>( c_{\text{max}}(n) )</td>
<td>1</td>
<td>1</td>
<td>2</td>
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Fig. 4. A digraph with four nodes and five arcs.

The result for \( n = 5 \) nodes is illustrated in Table 2 and Fig. 5 for digraphs with seven, eight and nine arcs.

<table>
<thead>
<tr>
<th>( m )</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
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<tbody>
<tr>
<td>( c_{\text{max}}(n) )</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
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Fig. 5. Digraphs with five nodes and seven, eight and nine arcs.

The generalized problem was attacked by Beineke & Harary (n.d.) who were able to obtain a recursive formula for the maximum number of triples in a digraph (\( c_{\text{max}}(n) \)) when there are \( m \) arcs and the number of nodes is suffi-
ciently large compared to \( m \). Their result determines \( c_{\text{max}}(n) \) exactly just when \( n \geq \sqrt{3}m \). This is illustrated in Table 3 for the pairs \((n, m)\).

**Tabl. 3.** The Beineke-Harary values for the exact number of cyclic triples \((c_{\text{max}}(n))\) with \( m \) arcs and \( n \geq \sqrt{3}m \) nodes.

<table>
<thead>
<tr>
<th>( m )</th>
<th>3</th>
<th>4</th>
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<tr>
<td>( n \geq \sqrt{3}m )</td>
<td>3</td>
<td>4</td>
<td>4</td>
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<td>5</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>( c_{\text{max}}(n) )</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Their formula gives \( c_{\text{max}}(n) \) for \( m = 3 \) arcs and \( n \geq 3 \) nodes. When \( n = 3 \) we have \( c_{\text{max}}(n) = 1 \) from Fig. 3 and no matter how many additional nodes there are, we still get \( c_{\text{max}}(n) = 1 \).

Table 3 tells that given \( m = 4 \) arcs, the value of \( c_{\text{max}}(n) \) for \( n \geq 4 \) is known, and we see from Table 1 that we still have \( c_{\text{max}}(n) = 1 \). The reason is that starting with Fig. 3, adding one more node and one more arc cannot produce any more cyclic triples.

Table 3 also gives \( c_{\text{max}}(n) = 2 \) when \( m = 6 \) and \( n \geq 4 \); see Fig. 6 and Table 1.

![Fig. 6. Asymmetric digraphs with five nodes and six arcs.](image)

Let us now consider the Mamboru alliance system. According to Needham (1987: 188) it is "an example of the second simplest type of social structure conceivable", the simplest type being "symmetric prescriptive alliance based on two lines". Needham characterizes the Mamboru system as the "product of three principles":
2. Asymmetry: the superiority of wife-givers to wife-takers.
3. Intransitivity: the wife-givers of wife-givers are not themselves wife-giv-
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ers. Needham regards intransitivity as an "intrinsic property" of the alliance system.

Implicit in Needham's (1987: 177) table of marriage alliances is an oriented graph with 28 nodes and 116 arcs. According to the Beineke-Harary formula, the maximum number of cyclic triples, $c_{max}(n)$, in such a digraph is 232. The actual number of cyclic triples in this digraph is only 51. This is much smaller than might be expected from Parkin's neutralization of transitivity hypothesis.

2. Genesis of transitivity. Parkin supposes that in an asymmetric prescriptive alliance system transitivity will be avoided through the formation of cycles of three intermarrying groups. We should therefore expect to find more cyclic than transitive triples in the digraphs of such systems (provided they are not tournaments). In the Mamboru alliance system there are, as mentioned, 51 cyclic triples but there are 141 transitive triples. Transitivity may not be a principle of Mamboru ideology but it is nonetheless an inherent property of the alliance system. There may be two reasons for the occurrence of transitivity in systems of this type.

The first is that the avoidance of transitivity is not an issue in the minds of the participants. It may be that they are basically concerned with dyadic relations between wife-givers and wife-takers and are indifferent to larger structures. Barnes has made this general point in his discussion of Eastern Indonesian systems: "Although in terms of empirical groupings, as well as ideological categories, asymmetric alliance produces triads and other plural patterns, the alliance tie is basically dyadic, that is, wife-giver versus wife-taker. Native language idiom expressing alliance is also commonly dyadic" (Barnes 1985: 98). This is not to say that the alliance system itself, whatever the native perception of it, is not in some sense circular. As Lévi-Strauss observed in connection with Leach's denial of circularity in the Kachin system, "For a matrilateral system to be devoid of circularity would require the number of local groups to be infinite [...] Effectively, the circularity of asymmetric systems does not arise from a pre-ordained disposition of local groups in unchanging cycles of exchange, but from the fact that, in whatever fashion they forge relationships between themselves, the genealogical space in which they move is structurally curved" (Lévi-Strauss 1987: 130).

On the other hand there are societies in which marriage alliance is thought of as fundamentally cyclic. Needham (1987: 168-169) reports that "Mamboru elders who explained the operation of the alliance system spontaneously repre-

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5. Matrix methods for finding the number of transitive triples and the transitive ratio of a digraph as described further on are given in Harary & Kommel (1979) and illustrated in Hage & Harary (1983).

6. Unfortunately, it is not possible to give an exact analysis of the cyclic and transitive properties of the Purum marriage system using the available data. Needham (1958) gives a "table of alliances" between Purum descent groups but he cautions that most of the marriage cycles are between non-localized groups: "An analysis of alliance cycles as they in fact link such local groups [...] would therefore be far more complex than the situation as I have analyzed it here" (ibid.: 82). Many authors have ignored this warning as Wilder (1971) has pointed out.
sented it as cyclic". Interestingly enough, Mamboru were "apparently quite excited intellectually by the task of finding cycles with the shortest path, i.e. such as were composed of just three terms".

The second reason is that transitivity occurs as a consequence (perhaps unintended) of shorter cycles being opened up in longer cycles of marriage exchange. The creation of a 3-cycle in a 4-cycle necessarily produces a transitive triple. The addition of any diagonal arc to the digraph of a 4-cycle will create a transitive triple along with the cyclic triple as can be seen by drawing such an arc in Fig. 7.

![Fig. 7. A 4-cycle. The addition of any diagonal arc will create one transitive triple and one cyclic triple.](image)

In Lévi-Strauss’s original formulation the opening up of shorter cycles is one of the factors leading to inequality and instability in a system of generalized exchange. Such a system presupposes equality by connecting wife-giving lineages in a single, short cycle of exchange—in the simplest case a 3-cycle. But "... the speculative character of the system, the widening of the cycle, the establishment of secondary cycles between certain enterprising lineages for their own advantage, and, finally, the inevitable preference for certain alliances, resulting in the accumulation of women at some stage of the cycle, are all factors of inequality, which may at any moment force a rupture" (Lévi-Strauss 1969: 266).

3. Degree of transitivity. If most asymmetric prescriptive systems (or systems of generalized exchange) have transitive substructures it is fair to ask just how transitive they are. The transitivity ratio of a digraph $D$ is the probability that if there is a 2-path in $D$, say from $u$ to $v$, then the arc $uv$ is also in $D$ (Harary & Kommel 1979; Hage & Harary 1983). Thus in picture writing we have

\[
\text{transitivity ratio of } D = \frac{\text{number of}}{\text{number of}}
\]
The transitivity ratio of the Kachin digraph (the tournament T5 in Fig. 2) is \(5/10 = 0.5\). Hence, the Kachin alliance system is just as transitive as it is cyclic. The transitivity ratio of the Mamboru digraph in Needham’s table of marriage alliance is \(141/575 = 0.25\). Contrary to Needham’s (1987: 191) assertion, intransitivity is not an “inherent property of the Mamboru alliance system”.

4. Ladder and Circle Societies. The contrast Parkin makes between ladder and circle hierarchies is an interesting one but it may be overstated. In ladder societies such as North India marriage is hypergamous and the “choice of spouse is directed not by kin category (except negatively, through incest rules) but by considerations pertinent solely to the hierarchy that exists outside the marriage system” (Parkin 1990: 477). In circle societies, as found in Southeast Asia and Eastern Indonesia, marriage is hypogamous and prescriptive. Contrary to Parkin’s model there are societies in which marriage is hypergamous and prescriptive in the sense that there is a rule of matrilateral cross-cousin marriage which maintains fixed alliances between lineages in successive generations. An example is Tongan society in West Polynesia which is, in many significant respects, the structural dual of Kachin society in Highland Burma (Mabuchi 1960; Hage & Harary 1996).

Tonga, like Kachin, was a stratified conical clan society (Gifford 1929; Friedman 1981; Hage & Harary 1996) except that lineage rank was based on a rule of primogeniture rather than ultimogeniture. As in Kachin there was a rule of matrilateral cross-cousin marriage but it was hypergamous rather than hypogamous. In Tonga this rule was institutionalized in that ego was expected to take a wife from (as well as appropriate the property of) a lineage or line to which it stood in the relation of fahu, literally, “above the law”. In Kachin and Tonga the form of anisogamy was consistent with political rank. In Kachin, as Leach pointed out, “Matrilateral cross-cousin marriage is [...] a correlate of a system of patrilineal lineages rigged into a class hierarchy. It does not necessarily follow that the bride-givers (mayu) should rank higher than the bride-receivers (dama); but it does follow that if class difference is expressed by marriage, then mayu and dama must be exclusive and one of the two must rank above the other” (Leach 1954: 256). In Tonga senior “elder brother” lines stood in the relation of wife-taker to junior “younger brother” lines. We note that Gifford (1929: 26) also saw a connection between lineage hierarchy and asymmetric marriage alliance: “It is quite possible that cross-cousin marriage in Tonga is genetically connected with the fahu concept, for it is usually the mother’s brother’s daughter whom a man marries. In other words, he may make demand in his capacity of fahu.”

In both Tonga and Kachin, matrilateral cross-cousin marriage was restricted mainly to persons of chiefly status. In Tonga, “The institution of fahu is established in the lower strata of society as well as in the higher, but it reaches its most extravagant development among the higher chiefs” (Gifford ibid. : 24).
"[Matrilateral cross-cousin] marriage was regarded as a chiefly prerogative not to be exercised by commoners" (ibid.: 281). For higher chiefs fahu marriages established and maintained political alliances. In Kachin (gumsa) society matrilateral cross-cousin marriage was similarly restricted. According to Leach, "It is really only persons of high status, such as the sons of chiefs and the sons of lineage heads, who need to conform strictly to the mayu-dama rules. Their marriages become thereby marriages of state. In the Hpalang situation, when the Nmwe and the Laga lineages declared that they were mayu-dama, this implied a relationship which in theory should continue indefinitely. But it did not imply an exclusive relationship such that all Laga males must marry Nmwe females. The relationship was adequately preserved as long as, in each generation, there was at least one marriage which conformed to the formal rule" (Leach 1954: 77).

In Tonga, as in Kachin, the solution to a bottleneck at the top of the marriage alliance hierarchy, produced by unmarriageable princesses rather than unmarriageable princes, was to find a spouse from another chiefly domain. Early in its history the Tongan solution to the problem of hypergamy was to make the (eldest) sister (the Tu‘i Tonga Fefine) of the paramount chief (the Tu‘i Tonga) a sacred virgin. But later the solution was that of “ethnic exogamy” (Rogers 1977) whereby the Tu‘i Tonga Fefine married into a Fijian chiefly house, the Fale Fisi, which stood outside the Tongan system of rank. This solution actually involved three parties linked in a “cousin circle path”.

Originally, Tonga was ruled by a single paramount chief, the Tu‘i Tonga. But around 1470 A.D. chiefly rule was divided between two collaterally related senior and junior lines of the conical clan into a sacred office, occupied by the Tu‘i Tonga, and a secular office, occupied at first by the Tu‘i Ha‘atakalaua and later, around 1610 A.D., by the Tu‘i Kanokupolu. A marriage connubium was established in the following way. Each successive chief of the Tu‘i Tonga line took as his “great chief wife” (moheofo) the (eldest) sister of the Tu‘i Kanokupolu chief and gave his own (eldest) sister to a chief of the Fale Fisi. The daughter of this marriage, the Tamaha, married a Tu‘i Kanokupolu chief elevating his prestige but not his political rank. Bott (1981: 56) describes the first two marriages as prescribed and the third as the result of “pressure to marry in a circle”. All three chiefly lines were thus united in a matrilateral connubium, but unlike the Kachin “cousin circle path” this did not equalize their status since Fijians did not count in the Tongan system of rank. Hence the Tongan marriage system was cyclic while the status system, consisting of a single arc, was vacuously transitive.

7. A third Polynesian solution to the problem of hypergamy was brother-sister marriage as in Hawaii (Rogers 1977; Hage & Harary 1991).
8. In a transitive relation the existence of a 2-path from \( u \) to \( v \) implies the existence of a 1-path as well. If there are no 2-paths in \( R \), the relation is said to be vacuously transitive.
Conclusion

Parkin (1990: 487) proposes that the formation of alliance cycles in asymmetric marriage systems ultimately “neutralizes”, “mitigates”, or “suppresses entirely” alliance asymmetry, i.e. transitivity, and that this “effectively disproves Lévi-Strauss’s view of the inherent instability of generalized exchange”. In graph theoretic terms this means that in an oriented graph the formation of cyclic triples precludes the formation of transitive triples. We have shown that this proposition has neither logical nor empirical support.

Logically, it follows from Kendall and Smith’s theorem that any asymmetric marriage system having the structure of a tournament and consisting of four or more wife-exchanging groups cannot be perfectly cyclic. A tournament with four nodes has at most two cyclic triples and two transitive triples and is therefore just as transitive as it is cyclic. The proportion is the same for a tournament with five nodes such as the digraph of the Kachin marriage system. In general, the proportion of cyclic triples decreases as the number of nodes increases approaching the limit of 1/4. In other words, an asymmetric marriage system that has the structure of a tournament becomes more transitive as it increases in size. In this respect Lévi-Strauss’s theory that the potential for inequality in an asymmetric marriage system increases as the alliance cycle expands and secondary cycles develop is mathematically correct. It also follows from this theorem that any asymmetric marriage system that is not a tournament but that contains as a subgraph a tournament with four or more nodes cannot be perfectly cyclic.

Beineke and Harary’s generalization of Kendall and Smith’s result gives an upper limit on the maximum number of cyclic triples in oriented graphs (not necessarily tournaments). An examination of the Mamboru alliance system, a “circle system” in Parkin’s terms, reveals that the actual number of cyclic triples is far less than the maximum number possible and is much smaller than the number of transitive triples. Transitivity in this system is not neutralized by the formation of cyclic triples.

In response to Leach’s (1961) criticism that, in the Kachin case, he failed to distinguish adequately between hypergamy and hypogamy, Lévi-Strauss observed that “from a formal point of view it is unnecessary to make the distinction between the two forms. We therefore proposed that in order to designate marriage between partners of unequal status—without concerning ourselves with whether it is the man or the woman who occupies the more elevated rank—we should borrow from botany the term anisogamy, which does not pre-judge the orientation of the system” (Lévi-Strauss 1987: 131). Since Parkin’s proposal is based on a presumed fundamental difference between hypergamy and hypogamy he finds Lévi-Strauss’s observation “extraordinary” (Parkin 1991: 348).
From a formal, mathematical, point of view it is of course unnecessary to distinguish between hypergamy and hypogamy because their digraphs are isomorphic. From a formal, ethnological, point of view a preconceived insistence on the radical difference between hypergamy and hypogamy may well obscure significant similarities between alliance systems. The Kachin system, even though it is based on hypogamy and ultimogeniture, is stratified in a way similar to the Tongan system which is based on hypergamy and primogeniture. It is significant that when Leach was casting about for a system like Kachin, he looked to Polynesia. If he had only read Gifford’s monograph on Tonga with its discussion of lineage rank and matrilateral cross-cousin marriage he would have found an analogue to Kachin (Hage & Harary 1996). In this connection it seems perfectly logical that Mabuchi (1960) would compare, as two related types of Malayo-Polynesian social structure, the “Oceanian” type as found in West Polynesia, and the “Indonesian” type of which Kachin is “reminiscent”. This is an extraordinary idea which could serve as a framework for the comparative study of Austronesian marriage systems.

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RÉSUMÉ

Per Hage & Frank Harary, La structure logique des systèmes de mariage asymétrique. — La structure logique des systèmes de mariage asymétrique est analysée en fonction de quatre questions. 1) Dans quelle mesure la formation de cycles peut-elle neutraliser la transitivité (hiérarchie) ? 2) Comment sont générées les structures transitives dans les systèmes cycliques ? 3) Comment ces systèmes peuvent-ils être comparés selon leurs degrés de transitivité ? 4) Les systèmes hypergamiques sont-ils nécessairement non prescriptifs ?

RESUMEN

Per Hage y Frank Harary, La estructura lógica de los sistemas de matrimonio asimétrico. — Se analizará la estructura lógica de los sistemas de matrimonio asimétricos en función de cuatro cuestiones: 1) En qué medida la formación de los ciclos puede neutralizar la transitividad (jerarquía) ? 2) Como se generan las estructuras transitivas en los sistemas cíclicos ? 3) Como pueden ser comparados estos sistemas según sus grados de transitividad ? 4) Son necesariamente non prescriptivos los sistemas hipergámicos ?